

$$\frac{W_2}{W_3} = \frac{3 M^2 (M - 1)^2 + (M - 1) \chi_2}{3 M^2 (M - 1)^2 - (3 M + 1) M \chi_2} \dots \dots \dots (10f)$$

$$\left. \begin{matrix} \chi_1 \\ \chi_2 \end{matrix} \right| = 4 M^2 - M + 1 \pm \sqrt{7 M^4 + 10 M^3 - 2 M + 1} \dots \dots (10g)$$

and

$$M = k^2 \dots \dots \dots (10h)$$

For  $k = 1$ , corresponding to a hypothetical cylinder of zero thickness, the ellipse is reduced to a line (zero surface) along  $p'_1$ . For  $k = \infty$  corresponding

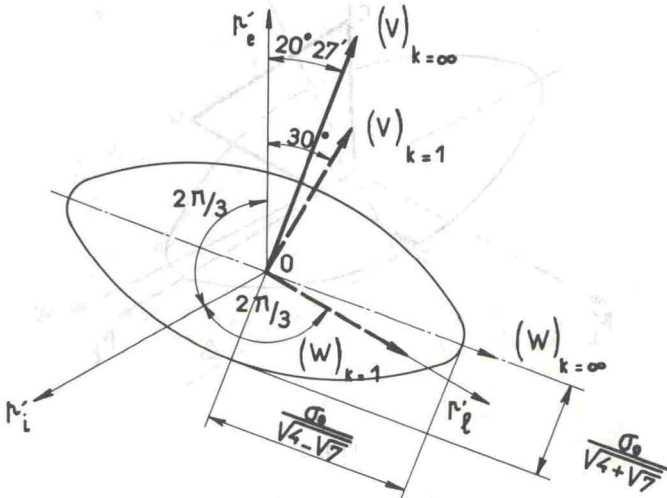


FIG. 2

to a cylinder of infinite thickness or to a capillary tube, the ellipse has the dimensions indicated in Fig. 2.

If, in the pressure space, the load is represented by the vector,  $\vec{OP}$ , with components  $p_i, p_1, p_e$ , the following remarks can be made:

1. Plastic flow is only possible if P lies on the elliptic cylinder;
2. since hydrostatic load is represented by a vector parallel to Z, only the component of  $\vec{OP}$  in the plane,  $\pi$ , is necessary for determining whether the material does or does not remain elastic; and

3. the ellipse corresponding to  $k = \infty$  having a finite dimension confirm the known result that a finite state of load is sufficient to create a plastic deformation in a cylinder of infinite thickness.

The second statement leads to the establishment of a graphic method permitting the resolution of the problems relative to elastic loading. On a first graph, A, three equidistant axes  $p'_1$ ,  $p'_1$ ,  $p'_e$  are traced, as well as the axes, V, for different values of K. For the same values a series of graphs, B, are traced on transparent paper representing the corresponding ellipses. The number of these graphs is limited both by the allowed interpolations and the fact that  $k = 4$  constitutes a limiting value in practice. Finally a new simpli-

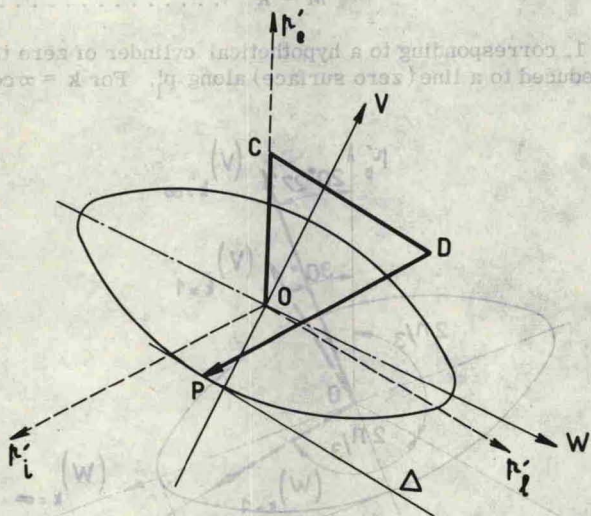


FIG. 3

fication is obtained by scaling the design to  $\sqrt{3/2}$  and on letting  $\sigma_0 = 1$ . Then the graphs are superimposed, A on B, making the axes V coincide, and tracing the projection of  $\vec{OP}$  on the plane  $\pi$  whose components on  $p'_1$ ,  $p'_1$ ,  $p'_e$  are respectively  $p_1/\sigma_0$ ,  $p_1/\sigma_0$ ,  $p_e/\sigma_0$ . The cylinder does or does not remain elastic according to whether P falls inside or on the ellipse.

With this method it is possible to find graphically, for a given value of one of the three variables, the possible maximum of one of the other two and the corresponding value of the third. Some of these results are well known. For example, it is shown in Fig. 3, how for a given value of  $p_e$ ,  $p_1$  could be determined so that  $p_1$  is a maximum. Beginning as before,  $OC = p_e/\sigma_0$  and  $\Delta$  are